Ferrimagnetic Resonance in Some Polycrystalline Rare Earth Garnets*

G. P. RODRIGUE†, J. E. PIPPIN†, W. P. WOLF†, AND C. L. HOGAN†

Summary—Ferrimagnetic resonance measurements have been carried out on a series of polycrystalline garnets of composition $5Fe_2O_3 \cdot 3M_2O_3$ with M = Y, Sm, Gd, Dy, Ho, Er, and Yb. These measurements were made over a temperature range from 20°C to the Curie points (approximately 280°C). The variations of line widths and effective g values over this temperature range are reported. Y, Yb, and Sm garnets have g values of approximately 2.0 at room temperature while those of Dy, Ho, and Er are appreciably less than 2.0. High-density yttrium garnet has a line width of approximately 50 oersteds at room temperature; line widths of other members of this series were found to vary from 400 to greater than 3000 oersteds. The effective g value and line width of the gadolinium garnet tend to very high values as its compensation point (17°C) is approached. The narrow line width of the yttrium garnet is found to depend strongly on the density of the sample. When the density decreases from 96 per cent to approximately 92 per cent of the theoretical value, the line width increases from 50 to about 150 oersteds. Several technical applications in which these materials might be particularly advantageous are discussed briefly.

Introduction

THE CLASS of ferrimagnetic oxides containing iron and rare earths and having the garnet crystal structure comprise a new series of compounds similar to ferrites but different in several important respects. They were discovered by Bertaut and Forrat[1] in the course of their work on ferrites containing rare earths, and they were also identified independently by Gilleo and Geller.[2] These compounds first aroused interest because of their rather unusual saturation magnetization vs temperature curves, which were first reported by Pauthenet[3] and are shown in Fig. 1. It will be seen that several of them have a "compensation point" at which the magnetization is zero, and that all of them have rather low absolute values of M_s above room temperature. This latter fact, coupled with their moderately high Curie temperature of approximately 280°C, immediately suggests the importance of these compounds in low-frequency microwave applications. However, for a material to be useful in this connection it is also necessary that it have a reasonably low resonance g value and, more important still, a narrow line width.

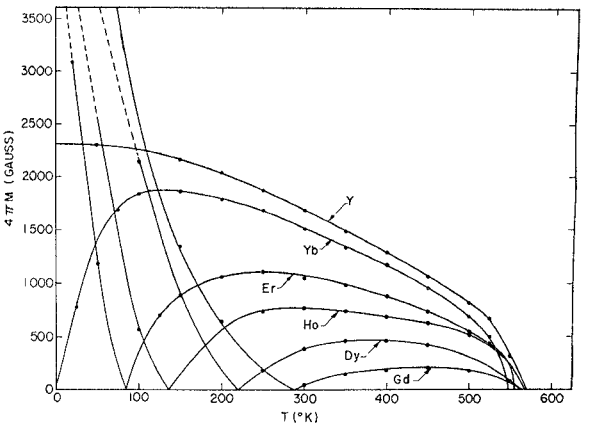

Fig. 1—Saturation magnetization vs temperature curves for $5Fe_2O_3 \cdot 3M_2O_3$ with M = Y, Yb, Er, Ho, Dy, and Gd (after Pauthenet).

In this paper we describe experiments carried out on seven different polycrystalline garnet materials to measure these two quantities from room temperature up to the Curie point.

The only previous resonance measurements reported in the literature are those of Dillon[4] on yttrium iron garnet single crystals, and Paulevé[5] on the polycrystalline gadolinium compounds. Dillon's measurements excited a great deal of interest since he reported resonance lines only a few oersteds wide, far narrower than for any ferrites previously examined. Furthermore, he found a rather low value for the anisotropy at room temperature and above, K_1/M ranging from 45 to 2 oersteds. One might expect therefore that even in the polycrystalline material line widths considerably narrower than those of ferrites might be obtained, especially if high-density materials could be made. This we have found to be the case, the line width for a sample of 96 per cent of theoretical density ranging from 50 oersteds at room temperature to 13 oersteds at 270°C. On other garnets nothing has been published concerning either the anisotropy or the natural line width. Our measurements show that none of the six other compounds we have examined are as promising as that containing yttrium, although they may prove to be useful as minor additions to the yttrium garnet.

There are several basic differences between the garnet crystal structure and the spinel structure of the ferrites. In the garnet there are three types of lattice sites for the metal ions; two of them, the 16a and the 24d are oc-

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[1] F. Bertaut and F. Forrat, "The structure of ferrimagnetic ferrites of the rare earths," *Compt Rend* , vol. 242, pp. 382–384; January, 1956.

[2] S. Geller and M. A. Gilleo, "The crystal structure and ferrimagnetism of yttrium-iron garnet," *Acta Crystall.*, vol. 10, p. 239; March, 1957, and private communications to be published.

[3] R. Pauthenet, "Magnetic properties of rare earth ferrites $5Fe_2O_3 \cdot 3M_2O_3$ with M = Tb, Dy, Ho, Er, Tm, Yb, Lu—experimental results," *Compt. Rend.*, vol. 243, pp. 1499–1502; November, 1956.

[4] J. F. Dillon, Jr., "Ferrimagnetic resonance in yttrium iron garnet," *Phys. Rev.*, vol. 105, pp 759–760; January, 1957.

[5] J. Paulevé, "Ferrimagnetic resonance in Gd garnet at 9300mc," *Compt. Rend.*, vol. 244, pp. 1908–1910; April, 1957.

cupied by Fe^{3+} ions and the other, the 24*c*, by the rare earth or yttrium ions. The magnetization curves shown in Fig. 1 can be explained[6,7] on the basis of Néel's theory, by postulating a strong antiparallel interaction between the ions on the 16*a* and 24*d* sites, and a weaker, also antiferromagnetic, coupling between the 24*c* and 24*d* sites. The resultant magnetization of the iron ions on the *a* and *d* sites is thus antiparallel to that of the rare earth ions on the *c* sites, and compensation points occur at temperatures where the magnitudes of the two magnetizations become equal. In the yttrium compound this complication does not arise since the trivalent yttrium ion has no magnetic moment so that the magnetization arises from the iron ions alone. A detailed description of the structure and magnetostatic properties of the garnets has been given in a series of articles by the workers at Grenoble[1,3,6–9] and also at the Bell Telephone Laboratories.[2,10] The fact that all the metallic ions in the garnet are trivalent suggests another important difference between garnets and ferrites. In ferrites the simultaneous presence of ions of the same element in different valence states greatly facilitates electrical conduction and hence leads to losses in high-frequency applications. Since ferrites invariably contain metal ions of different valencies, usually 2 and 3, it is often necessary to strike a compromise in the preparation process between keeping the one type of ion oxidized and the other reduced. This is sometimes difficult although a great deal can be done by the inclusion of certain additives (*e.g.*, manganese). In garnets, on the other hand, all metal ions present are trivalent (which is normally their state of highest oxidation). Thus no compromise need be made in the firing conditions, and one would expect extremely low electrical losses. The loss tangents for our various samples shown in Table I, opposite, indicate that this is indeed the case, when it is remembered that these were "pure" stoichiometric samples, free from any beneficial additives.

The other remarkable feature of the garnet structure is that all of the *a*, *c*, and *d* sites allowed for metal ions are filled, as compared with ferrites in which only half the *A* and *B* sites are randomly filled. This fact is important in connection with the mechanisms giving rise to the "natural" ferrimagnetic resonance line width,[11] as well as in aiding the interpretation of the other basic physical properties.

[6] R. Pauthenet, "Interpretation of the magnetic properties of ferrites $5Fe_2O_3 \cdot 3M_2O_3$ with M = Y, Gd, Tb, Dy, Ho, Er, Tm, Yb, Lu," *Compt. Rend.*, vol. 243, pp. 1737–1740; November, 1956.

[7] L. Néel, "On the interpretation of the magnetic properties of rare earth ferrites," *Compt. Rend.*, vol. 239, pp. 8–11; July, 1954.

[8] R. Aléonard, J. Barbier, and R. Pauthenet, "Magnetic properties of yttrium ferrite $5Fe_2O_3 \cdot 3Y_2O_3$ of the garnet type," *Compt. Rend.*, vol. 242, pp. 2531–2533; May, 1956.

[9] R. Pauthenet, "Magnetic properties of gadolinium ferrites," *Compt. Rend.*, vol. 242, pp. 1859–1862; April, 1956.

[10] S. Geller and M. A. Gilleo, private communication to be published.

[11] A. M. Clogston, H. Suhl, L. R. Walker, and P. W. Anderson, "Ferromagnetic resonance line width in insulating material," *J. Phys. Chem. Solids*, vol. 1, no. 3, pp. 129–136; 1956.

Materials

The chemical formula for the ferrimagnetic garnets is $5Fe_2O_3 \cdot 3M_2O_3$ where M denotes a trivalent rare earth ion from samarium to lutecium, or yttrium. For the present investigation samples were prepared with M = Sm, Gd, Dy, Ho, Er, Yb, and Y, using the coprecipitation technique described elsewhere.[12] This method has the advantage of producing high-density, low-loss materials whose chemical composition can be accurately controlled even on a small scale preparation. X-ray powder diffraction patterns were taken on all the samples using a General Electric XRD5 spectrometer. No lines other than those characteristic of the garnet structure were found. The lattice constants determined from the patterns are in good agreement with those found by Bertaut, *et al.*, and by Gilleo and Geller, as seen in Table I. Measurements of magnetic moment at room temperature, kindly made for us by C. H. Nowlin using a pendulum magnetometer, yielded values of $4\pi M_s$ given in column 8. The values are given in Gauss adjusted to the theoretical X-ray density. Values of $4\pi M_s$ calculated from Pauthenet's published magnetization curves in Bohr magnetons/molecule are shown in column 9. It will be seen that there is some systematic discrepancy. This may be due in part to difficulty in reading values off Pauthenet's curves.

Experimental Procedure

Resonance measurements were made on small (0.015-inch to 0.035-inch) spherical samples at *X* band, using a reflection-type apparatus similar to that described by Artman and Tannenwald[13] and by Spencer, LeCraw, and Reggia.[14] To minimize wall effects the spheres were placed about $1\frac{1}{2}$ diameters off the center of the end wall of the rectangular cavity. The cavity was operated in the TE_{101} mode at a resonant frequency of 9208 mc at room temperature, decreasing by 40 mc as the temperature was raised to 300°C. The cavity was made of copper and continuously flushed with dry nitrogen gas to prevent corrosion at high temperatures. Temperatures up to 300°C were obtained by a small furnace completely surrounding the cavity. The temperatures were measured, with an accuracy of ± 3°C, by means of an iron-constantan thermocouple attached to the cavity wall.

The reflection coefficient of the cavity (and therefore the standing wave ratio in front of the cavity) was measured by sampling the direct and reflected signals and comparing them after detection, using two crystal detectors calibrated to agree within ± 0.05 db over the range of signal variations. A HP X382A precision cali-

[12] W. P. Wolf and G. P. Rodrigue, "The Preparation of Polycrystalline Ferrimagnetic Garnet Materials for Microwave Applications," Harvard Univ. Gordon McKay Lab., Cambridge, Mass., Contract AF 19(604)-1084 Sci. Rep. No. 9; also to be published.

[13] J. O. Artman and P. E. Tannenwald, "Measurement of susceptibility tensor in ferrites," *J. Appl. Phys.*, vol. 26, pp. 1124–1132; September, 1955.

[14] E. G. Spencer, R. C. LeCraw, and F. Reggia, "Measurement of microwave dielectric constant and tensor permeabilities of ferrite spheres," 1955 IRE Convention Record, pt. 8, pp. 113–121.

TABLE I

$3M_2O_3 \cdot 5Fe_2O_3$

M	Lattice Constant (in A°)	(Bertaut *et al.*)	Density (exp.) in gm/cc	Theoretical X-Ray Density in gm/cc	Per Cent Density	Dielectric Loss tan (at 20 mc)	$4\pi M_s$ (Gauss ±Per Cent)	$4\pi M_s$ (Gauss) (Pauthenet)	ΔH Oersteds at Room Temp.
Y	12.360±0.007	12.36	4.95	5.190	95.5	~0.003	1740±3.6	1680	50
Sm	12.505±0.005	12.52	5.11	6.265	82	~0.002	1695±5.4		2500
Gd	12.445±0.005	12.44	6.22	6.490	96	~0.0015	169±4.2	50	
Dy	12.385±0.005	12.38	6.08	6.701	91	~0.004	537±3.2	400	
Ho	12.350±0.005	—	6.05	6.811	89	~0.0015	976±3.6	780	
Er	12.330±0.005	12.33	6.33	6.892	92	~0.004	1308±3.6	1100	1400
Yb	12.283±0.005	—	6.08	7.097	86	~0.0025	1640±6	1500	530

brated attenuator in the direct arm was used to measure the change in reflected power with magnetic field. Its absolute accuracy, guaranteed by the manufacturer to be within 2 per cent, was found to agree well with a second attenuator of the same type. From the measured reflection coefficients it is possible to calculate the imaginary part (χ_{xx}'') of the diagonal component of the "external" susceptibility tensor, using standard Bethe-Schwinger perturbation theory. In mks units χ_{xx}'' is given by

$$\chi_{xx}'' = \frac{V}{4\Delta v}\left(\frac{\lambda_g}{\lambda_0}\right)^2 \frac{1}{Q_e}(\rho - \rho_0),$$

where

V = volume of the cavity,
Δv = volume of the sample,
λ_g = wavelength in the empty cavity,
λ_0 = free space wavelength,
Q_e = "external" or radiation Q of the cavity,
ρ = vswr in front of the loaded cavity at some applied H field,
ρ_0 = vswr in front of the empty cavity.

The estimated accuracy in the absolute values of χ_{xx}'' is approximately 20 per cent. Relative measurements, from which line width and effective g values are determined, are more accurate. Line widths are estimated to be accurate to 5 per cent except for the samples having extremely wide lines (Sm and Ho), for which the error may be as large as 10 per cent.

Because the magnetic field was measured with extreme accuracy by means of nuclear magnetic resonance, errors in g factor are due primarily to uncertainty in adjusting the magnetic field for maximum absorption, and thus they depend strongly on the line width of the particular sample. For yttrium garnet, with a line width of 50 oersteds, the error is about 0.2 per cent while the g factors of samarium and holmium garnets are accurate to about 2 per cent.

Results

Room Temperature Measurements

At room temperature, four of the seven samples gave a ferrimagnetic resonance absorption for magnetic fields in the range zero to 6100 oersteds. The absorption

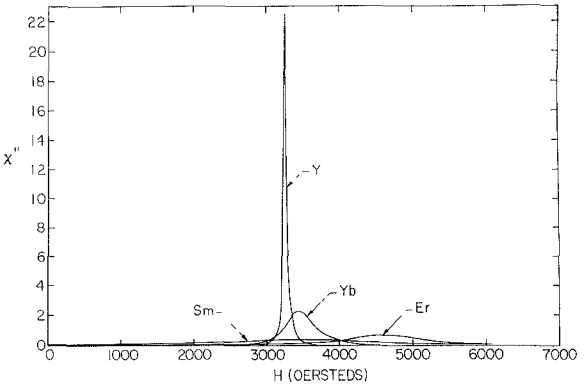

Fig. 2—Room temperature curves of χ_{xx}'' (mks values) vs H.

curves, expressed in absolute mks units of χ'' are shown in Fig. 2. It will be seen that the yttrium garnet has by far the narrowest line width (~50 oersteds) and correspondingly the largest value of χ''. For greater ease of comparing the line widths and resonance field values, these absorption curves were normalized to the same peak height; the results are shown in Fig. 3 where each line is compared to that of yttrium. The absence of resonance in the cases of the holmium and dysprosium garnets was explained by measurements made at higher temperatures (see below) which indicated that the resonance field at room temperature is greater than 6100 oersteds, our highest available field. Resonance in the gadolinium garnet was also observed at higher temperatures, and here the results indicated a trend towards broader and weaker lines as the magnetic compensation point at about 17°C was approached.

In order to detect the possible effect of sphere size on observed g factor and line width, spheres of at least two different diameters were tried for all samples. When the diameter was doubled from 15 to 30 mils, no appreciable effect on the g factor was observed beyond the limits of experimental error mentioned previously. This doubling of the sphere diameter increased the observed line width in all samples by roughly 10 per cent. The values quoted below refer to the smallest sphere in each case.

The effective g factors reported here were calculated from the well-known equation

$$\omega_0 = \gamma H_0 \qquad (1)$$

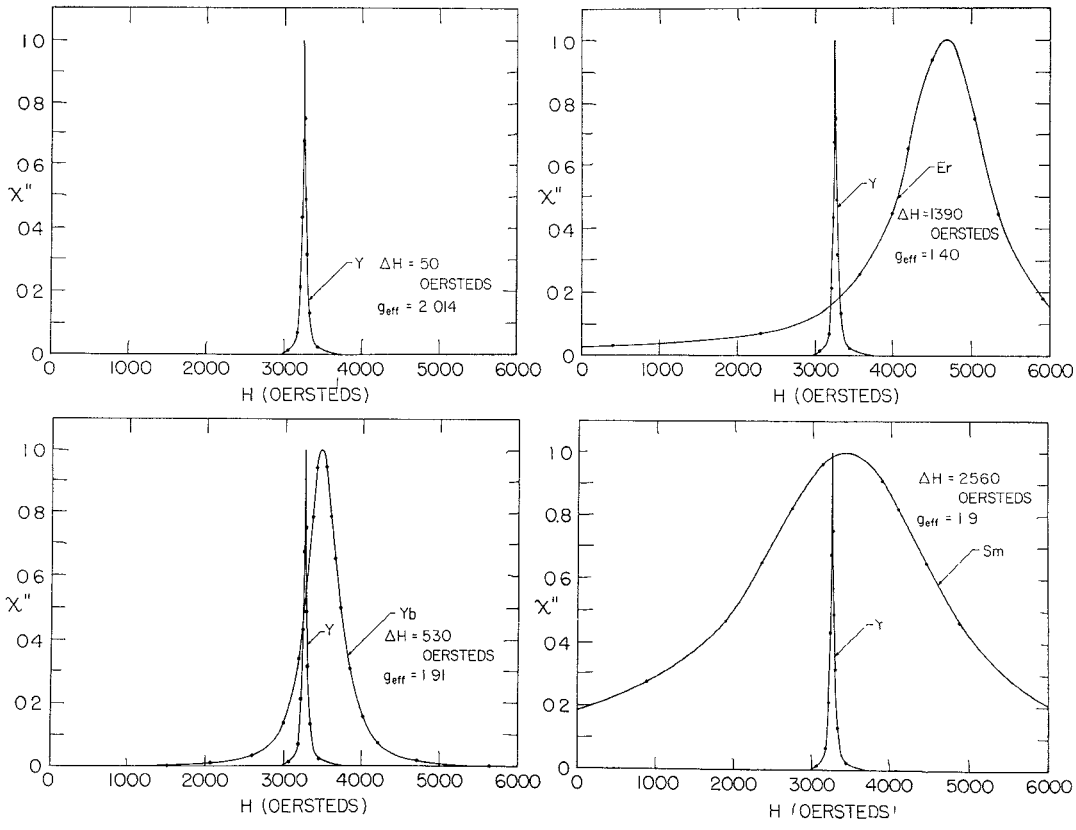

Fig. 3—Normalized room temperature curves of χ_{xx}'' vs H.

where $\gamma = g_{eff}(e/2mc)$, and H_0 is the resonance field. However, Okamura *et al.*[15] have observed that the effective g factor is frequency dependent, and they attributed this to the existence of an apparent internal field H_i. Thus the true g factor should be determined from

$$\omega_0 = g_{true}\frac{e}{2mc}(H_0 + H_i). \tag{2}$$

Kojima[16] and McGuire,[17] noting the effect of porosity on g factor, have suggested that this internal field must, in part at least, be dependent upon the density of the sample. This density effect and the shift in resonant field due to anistropy have been treated theoretically by Schlömann,[18] who has obtained for the internal field of (2) the expression

$$H_i = \frac{4\pi M_s}{3}\frac{v}{V} + \frac{1}{4}H_a, \tag{3}$$

where V is the total volume of the sample, v the volume of all pores, and $H_a = 2K_1/M$, K_1 being the first order anisotropy constant. As the density effect is directly proportional to the magnetization, it should for our samples be most important in the yttrium compound. Samples of yttrium garnets with densities varying from 81 to 96 per cent of the theoretical X-ray density were tested. Fig. 4 shows the agreement with Schlömann's theory. The experimental points for the internal field were calculated by combining (1) and (2) to give

$$H_i = \left(\frac{1}{g_{true}} - \frac{1}{g_{eff}}\right)\frac{f_0}{1.400}.$$

The true g factor was taken from Dillon's single crystal measurements at room temperature, and g_{eff} is that observed in samples of the various densities; f_0 is the frequency in mc. The large uncertainty at the lower densities is due to difficulties in obtaining true spheres from the lower density materials. The slope of the theoretical line was determined by the saturation magnetization, and the line was then drawn so as to obtain the best agreement with the experimental values. The intercept corresponding to 100 per cent density indicates a value for $\frac{1}{4}H_a$ of the order of 15 to 20 oersteds, in satisfactory agreement with Dillon's value. The densities are those measured on the gross sample and represent a minimum density of the sample. They may be in error by 3 per cent.

Schlömann[18] has also treated the effect of density on

[15] T. Okamura, Y. Torizuka, and Y. Kojima, "The g factor of ferrites," *Phys. Rev.*, vol. 88, pp. 1425–1426; December 15, 1952.
[16] Y. Kojima, "The g factor of ferromagnetic spinels," *Sci. Rep. Res. Inst., Tohoku University*, vol. A-6, pp. 614–622; December, 1954.
[17] T. R. McGuire, "The frequency dependence of g values in ferrites," *Proc. AIEE Conf. on Magnetism and Magnetic Materials*, pp. 43–46; 1955.
[18] E. Schlömann, "The Microwave Susceptibility of Polycrystalline Ferrites in Strong DC Fields and the Influence of Nonmagnetic Inclusions on the Microwave Susceptibility," presented at Conference on Magnetism and Magnetic Materials, Boston, Mass.; October, 1956.

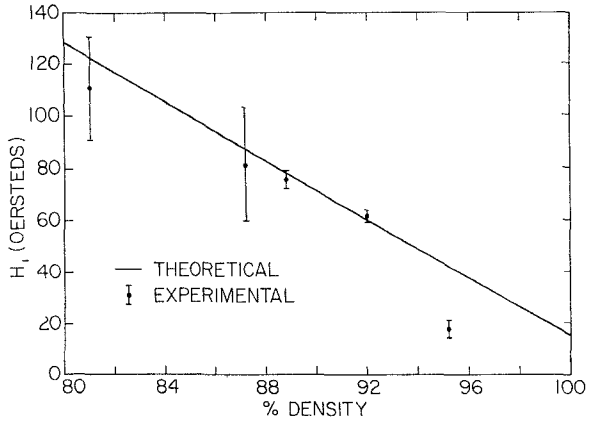

Fig. 4—Internal field of (3) as a function of density. Slope of theoretical line determined by $4\pi M_s = 1680$ Gauss. Best fit with experimental points obtained with $H_u \doteq 15$ oersteds.

line width. He suggests that the line broadening due to pores alone would be given by

$$\Delta H_{\text{pores}} = 1.5(4\pi M_s) \frac{1}{\frac{V}{v} + 1} \cdot \qquad (4)$$

This effect is observed to be of great importance in the yttrium garnet because of its extremely small intrinsic line width and low anisotropy field. Fig. 5 shows the experimentally observed line widths and the theoretical curve representing the line broadening due to pores *alone*, as calculated from (4). Again the densities may be in error by 3 per cent.

The close agreement suggests that in this material, in which K_1/M is known to be small compared with $4\pi M$, anisotropy contributes very little to the line width, in agreement with the recent theory of Geschwind and Clogston[19] on dipolar narrowing. It appears that the intrinsic line width of a 100 per cent dense polycrystalline yttrium garnet sphere at room temperature would be considerably narrower than the 50 oersteds reported here. It is obvious that to take full advantage of the narrow line widths in polycrystalline yttrium garnets extremely high densities must be obtained. In even a 93 per cent dense sample most of the line broadening is due to pores.

A close examination of the shape of the resonance curves shows that all of them are slightly asymmetric, particularly on the lower parts of the χ'' curves, as can be seen in Fig. 3 and Fig. 6. For Y, Gd, and Yb, the wings of the resonance curves are steeper on the low field side, while those of Er, Ho, and Sm are steeper on the high field side. If the polycrystalline line width were entirely due to anisotropy broadening this would indicate, according to Schlömann's theory,[18] that Y, Gd, and Yb have negative anisotropies while those of Ho, Er, and Sm are positive. However, if the line widths were entirely due to anisotropy, we would expect very

[19] S. Geshwind and A. M. Clogston, private communication.

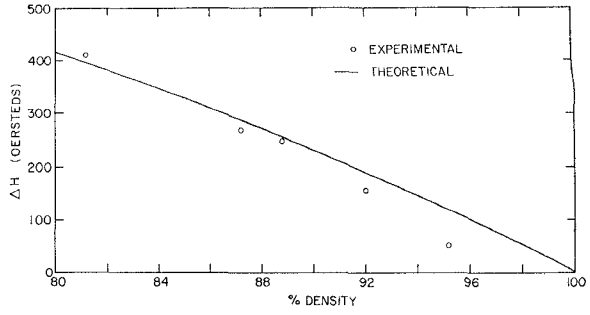

Fig. 5—Line width of yttrium garnet as a function of density. Theoretical curve calculated from (4).

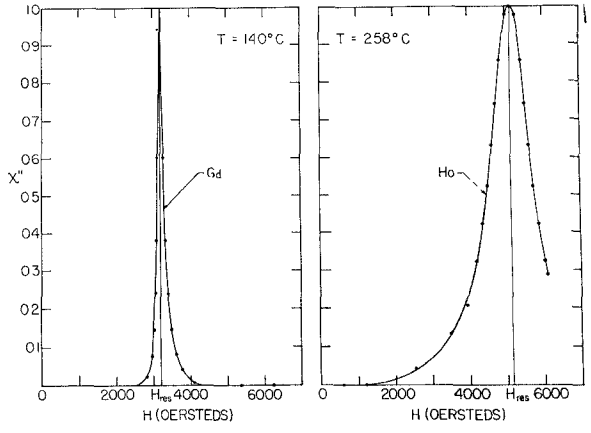

Fig. 6—Resonance lines in polycrystalline holmium and gadolinium garnets.

much larger asymmetries than those observed, and we must suspect therefore that there are other important contributing factors. In the yttrium garnet we know that porosity plays an important role in determining the line width while preliminary measurements on single crystals of the gadolinium and samarium garnets have shown that even single crystals have line widths very close to those reported here for the polycrystalline materials. It must be concluded, therefore, that there is no simple relationship between the asymmetry and the sign of the anisotropy in these materials, and in fact some recent measurements have shown that the samarium garnet has a negative anisotropy, with K_1/M of only about 120 oersteds. The line width of the samarium garnet single crystal was found to be about 2000 oersteds at room temperature, approximately independent of crystal orientation. Measurements on single crystals of gadolinium garnet by R. L. White (private communication) have shown that the line width depends markedly on crystalline orientation and varies from about 1000 oersteds at 20°C to about 140 oersteds at 90°C. K_1 was found to be negative.

High-Temperature Measurements

Ferromagnetic resonance measurements were made as a function of temperature on all samples from room temperature to the Curie points, which for all the seven materials are in the range of 265° to 280°C.

The variation of effective g factor with temperature

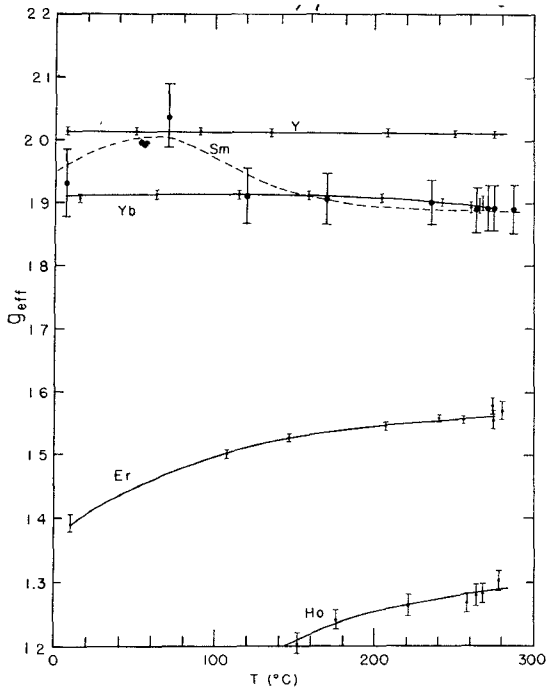

Fig. 7—Temperature variations of the effective g value in polycrystalline garnets.

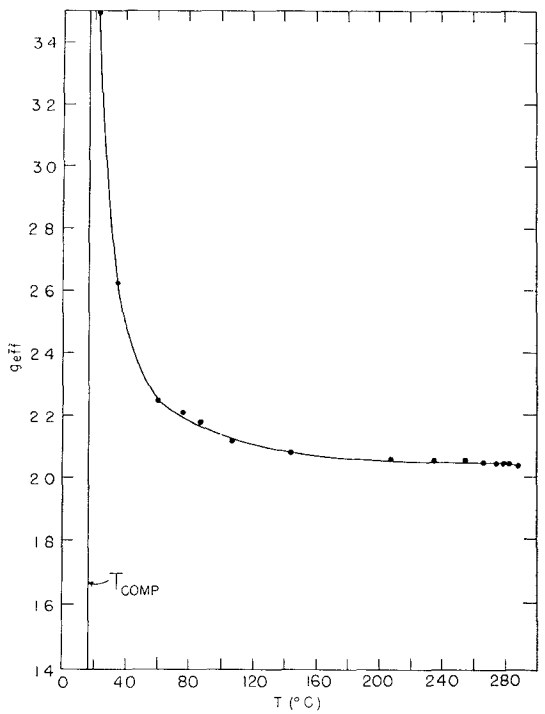

Fig. 8—Effective g value of gadolinium garnet as a function of temperature. ($T_{comp}=17$°C.)

for Y, Yb, Er, Ho, and Sm garnets is shown in Fig. 7, and for Gd garnet in Fig. 8. It will be seen that all of them with the exception of the yttrium compound have g factors which are temperature dependent and appreciably different from 2. There may be several factors contributing to this. As the temperature changes both the anisotropy and the magnetization change and hence the internal field H_i, given by (3), changes. Moreover, in all but the yttrium and gadolinium garnets there are ions having orbital angular momentum which will be partially quenched in the solid. The effect of the quenching may be expected to change with temperature and this will be reflected by a change in g factor. For the gadolinium garnet g moves towards extremely large values as the compensation point T_c, is approached. This has also been reported by Paulevé.[5] It is probably due to the combined effects of anisotropy shift and of the existence of slightly different compensation points for the total magnetization and the total angular momentum.

At high temperatures where the shift in g due to anisotropy and porosity is negligible, the g factor observed on the yttrium garnet ($g_{eff}=2.011\pm0.002$) agrees very well with that reported by Dillon[4] on a single crystal at the same temperature ($g=2.009\pm0.002$).

The peak of the holmium resonance curve was first observed below 6100 oersteds at about 140°C. The peak in the resonance line of the dysprosium garnet was not found below 6100 oersteds at any temperature, though the lower wing of the line was plainly observable at the highest obtainable fields at temperatures above 240°C. Thus its effective g factor remained less than 1.2.

The results of measurements of line width vs temperature for Yb, Er, Sm, and Ho garnets is shown in Fig. 9 and for yttrium and gadolinium in Fig. 10. The minimum line width on all curves occurs at temperatures slightly below the Curie point reported by Pauthenet.[3] The sudden increase in the vicinity of 270° to 280°C was observed in all samples. It is probably due to the combined effect of several causes: near the Curie point M tends to small values very rapidly while K_1 tends to zero more slowly.[20] Thus K_1/M would increase as M decreases. This would cause an increase in the line width both by increasing the usual anisotropy broadening and also by reducing the effect of dipolar narrowing as suggested by Geschwind and Clogston. Near the Curie point, even in single crystals, the effect of exchange interaction also becomes small and thus the usual exchange narrowing, which leads to lines considerably narrower than would otherwise be expected on the basis of magnetic dipole—dipole interaction—ceases to operate. A sharp increase in the line width of a single crystal has been observed by Dillon[21] in yttrium garnet near the Curie point. The narrowest line observed below the Curie point in the polycrystalline material was 13 oersteds for the yttrium compound at 272°C. It should be noted that a substantial part of this decrease from the value at room temperature is due to the lessening of porosity broadening as the magnetization decreases.

It can be seen from Fig. 9 and Fig. 10 that the curves of line width vs temperature for all the garnet materials studied have the same general shape, with the obvious

[20] W. P. Wolf, to be published.
[21] J. F. Dillon, Jr., private communication.

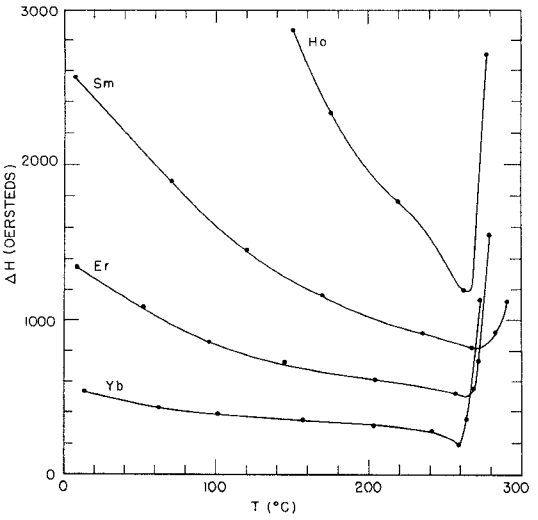

Fig. 9—Line width vs temperature in polycrystalline garnet materials.

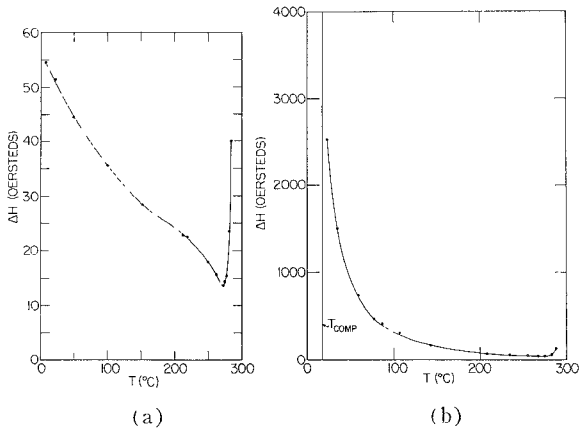

Fig. 10—(a) Line width of yttrium garnet as a function of temperature. (b) Line width of gadolinium garnet as a function of temperature.

exception of that of the gadolinium garnet, which has a compensation point near room temperature. The anomalous behavior of the gadolinium garnet in this range of temperature is of course to be expected, since the effective anisotropy field, $2K_1/M$, grows rapidly as M tends to zero at the compensation point. The recent experiments on single crystals quoted above have shown, however, that this is not the only cause for the increase in line width and that there is also a considerable contribution from the intrinsic line width of the individual crystallites.

A Discussion of Technical Applications

Because of their unusual properties the garnets offer unusual possibilities in low frequency microwave applications. Hogan[22] and Lax[23] have discussed the low-frequency problem in some detail.

[22] C. L. Hogan, "The low-frequency problem in the design of microwave gyrators and related elements," IRE Trans., vol. AP-4, pp. 495–501; July, 1956.

[23] B. Lax, "Frequency and loss characteristics of microwave ferrite devices," Proc. IRE, vol. 44, pp. 1368–1386; October, 1956.

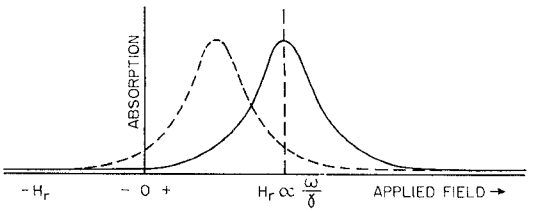

Fig. 11—Depicting absorption curve in an isolator neglecting all phenomena except ferromagnetic resonance. The dotted curve is for a lower frequency or higher γ.

We will discuss here the very important materials aspect of the low-frequency problem with the realization that certain design techniques can significantly modify any numerical results which are obtained. For simplicity the infinite medium theory as discussed by Hogan will be used. It does not apply quantitatively in waveguides, of course, but it is useful for comparison purposes and for helping to make the physical picture more concrete.

Consider the resonance absorption isolator. Fig. 11 depicts the absorption curve, neglecting all phenomena except ordinary ferromagnetic resonance. For positive circular polarization the absorption peak occurs at a field H_r which is proportional in some way (depending on the geometry) to ω/γ. For negative circular polarization the insertion loss is determined in part by the tail of the absorption line at $-H_r$. The narrower the line, the lower the absorption on the tail for a given H_r, and the higher the peak for a given magnetic moment. For lower frequency or higher γ the line moves toward lower fields and the back-to-front ratio decreases, *i.e.*, the isolator deteriorates. This simple physical picture demonstrates the desirability of low g values and narrow line widths at low frequencies. The g value of polycrystalline yttrium garnet is for practical purposes 2.0,[24] and the line width 50 oersteds at room temperature. These numbers show a striking advantage over ordinary ferrites with equivalent Curie temperature ($\sim$300°C), where one might expect line widths no smaller than about 200 oersteds. In even denser yttrium garnet the line width might be reduced to around 30 oersteds, as previously discussed, with the attendant improvement in low-frequency operation. Furthermore, some of the garnets have very low g values (Er, 1.4; Ho and Dy, 1.2).

The physical picture discussed here is informative, but the problem is more correctly treated by solving mathematically for the attenuation in terms of the intrinsic properties of the material. Hogan[22] has done this in terms of circularly polarized waves in an infinite medium, and he finds for the back-to-front ratio,

$$R = \sqrt{128\left(\frac{\omega}{\gamma \Delta H}\right)^3}, \tag{5}$$

[24] In the nickel ferrite-aluminates which have been developed for low-frequency applications (see Pippin and Hogan, this issue, p. 77) the g value is significantly greater than 2.

where ΔH is the line width. This can be rearranged to give

$$\omega_{\min} = \frac{\gamma \Delta H R^{2/3}}{5.0} . \tag{6}$$

This is the minimum frequency for a given back-to-front ratio. If a ratio of 10 is desired, this gives

$$\frac{\omega_{\min}}{\gamma} = 0.92 \Delta H. \tag{7}$$

Inserting $\gamma = 2.8$ mc/oersted and $\Delta H = 50$ oersteds,[25] the values for the yttrium garnet, one gets

$$f_{\min} \doteq 130 \text{ mc.}$$

By comparison a ferrite with $\Delta H = 200$ oersteds would give $f_{\min} = 520$ mc.

The narrow line width has been made to look attractive in this discussion; however, the more narrow line width materials tend to deteriorate more rapidly at high powers, and this can be a serious limitation. On the other hand, the narrow line yttrium garnet should prove extremely effective in all microwave devices designed to exploit nonlinear behavior, such as passive limiters. Others such as frequency doublers, mixers,[26] and detectors[27] depend on the ability of the magnetization vector to "fan out" in large precessional motion, and this motion is accentuated in narrow line width materials. With respect to the detector, magnetostrictive properties of the material are important; little, if anything, is known about magnetostriction in the garnets at this time.

While dealing fancifully with 130-mc isolators, we have neglected some important considerations at low frequencies. The material must be magnetized for the theory to apply, and to gain the most from the ferromagnetic absorption phenomenon. The resonance field for extremely low frequencies may be insufficient to magnetize the material unless care is taken in choosing the geometry. A field necessary for saturation is approximately equal to the demagnetizing field plus the anisotropy field and in this respect the yttrium garnet is favorable, having an anisotropy field of only 90 oersteds at room temperature. As an example, a long thin slab magnetized perpendicular to the plane of the slab has a resonance field given roughly by

$$H_r = \frac{\omega}{\gamma} + 4\pi M.$$

The field necessary to magnetize it is

$$H_{\text{mag}} = 4\pi M + \mathrm{H}_{\text{anis}},$$

and so if the resonance field is to magnetize the slab, then

$$\frac{\omega}{\gamma} \geq H_{\text{anis}} = \frac{2K_1}{M} .$$

For yttrium garnet, with $2K_1/M = 90$, this requires that

$$f \geq 250 \text{ mc.}$$

For comparison, nickel ferrite with $H_{\text{anis}} = 400$ and $g = 2.34$ would require

$$f \geq 1300 \text{ mc.}$$

Up to now the low field losses have been neglected. These losses occur because of domain structure in incompletely magnetized materials, and have been studied in the ferrites by Rado and others.[28] There is insufficient information to describe these losses quantitatively at present, and such losses have not yet been measured at all in the garnets. Nevertheless, it has been shown[29] that in general, zero field losses extend up to a frequency of

$$\omega_{\max} = \gamma(H_{\text{anis}} + 4\pi M_s)$$

and for yttrium garnet this gives $\omega_{\max} = 4900$ mc.

One way of lowering this limit which has been used in the past for ferrites is to reduce M_s by replacing some of the magnetic ions by aluminum. However, this also reduces the Curie point and there is a limit beyond which one cannot go in practice. Such considerations would not arise if one were to use instead an yttrium garnet with some of the yttrium ions replaced by rare earth ions with large magnetic moments (*e.g.*, Gd or Tb), since the Curie point is almost the same for all the rare earth garnets. With such a mixed garnet any value of M_s between zero and the value for the pure yttrium compound could be obtained.

Another way in which mixed garnets might become technically useful is as materials whose magnetization is almost constant over a certain range of temperatures. It can be seen from Fig. 1 that M increases with T over a considerable range of temperatures for all compounds having compensation points, while for the yttrium (and lutecium) garnets it decreases steadily with T. Thus it is easy to envisage a mixed compound whose M vs T curve would have a stationary point at any desired temperature below the Curie point and relatively little variation on either side.

The low field losses, occurring in unmagnetized media, are completely reciprocal with respect to wave propagation. This is another strong reason for having the ferrite saturated by the resonance field in an isolator; else the low field losses will produce attenuation in the forward direction and decrease the isolation ratio. This is illustrated in Fig. 12 in which the low field losses and ferromagnetic resonance losses are shown separately. Gen-

[25] This assumes the line width does not change with frequency.

[26] W. P. Ayres, P. H. Vartanian, and J. L. Melchor, "Frequency doubling in ferrites," *J. Appl. Phys.*, vol. 27, pp. 188–189; February, 1956. Also, J. E. Pippin, "Frequency doubling and mixing in ferrites," PROC. IRE, vol. 44, pp. 1054–1055; August, 1956.

[27] D. Jaffe J. C. Cacheris, and N. Karayianis, "Ferrite microwave detector," 1957 NATIONAL IRE CONVENTION RECORD, pt. 1, pp. 242–249.

[28] G T. Rado, R. W. Wright, and W. H. Emerson, "Ferromagnetism at very high frequencies. III. Two mechanisms of dispersion in a ferrite," *Phys. Rev.*, vol. 80, p. 273; October 15, 1950.

[29] L. G. Van Uitert, J. P. Shafer, and C. L. Hogan, "Low-loss ferrites for applications at 4000 mc/sec," *J. Appl. Phys.*, vol. 25, pp. 925–926; July, 1954.

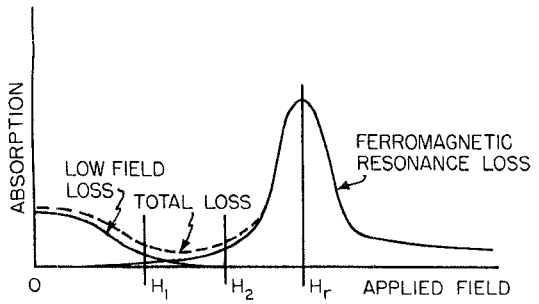

Fig. 12—The effect of low field loss on total loss in an isolator. A phase shifter would operate between H_1 and H_2.

erally the low field loss is reduced in magnitude when the saturation moment is decreased;[30] for this reason it is often desirable to have a low saturation moment even in an isolator. The moments of the garnets are relatively low.

For nonreciprocal phase shifters operating below the field required for resonance, the low frequency problem is even more difficult. Generally such phase shifters operate in the region of minimum attenuation, where the low field losses have nearly disappeared and the resonance losses have not yet become large (for example, between H_1 and H_2 of Fig. 12). For low frequencies the minimum may be very narrow or even nonexistent, as the absorption line gets closer to zero field. Then it becomes necessary to reduce the saturation moment until the low field losses do not occur. Fig. 12 also demonstrates the desirability of a narrow line width and low g factor for phase shifters.

[30] This is true for sufficiently high frequencies. For lower microwave frequencies (~1000 mc) the problem is more complicated.

Using infinite medium theory, Hogan has calculated the differential phase shift per db loss for such a phase shifter, neglecting the low field loss and including only the ferromagnetic resonance loss due to the tail of the absorption line. He finds

$$\theta/L_+ = \frac{\omega}{2.2\gamma\Delta H},$$

where θ is differential phase shift and L_+ is the attenuation of the positive circularly polarized wave. If a differential phase shift of $\pi/2$ radians with 0.5 db loss is desired, then for the yttrium garnet

$$\frac{\omega}{\gamma} \geq 380$$

or

$$f \geq 1050 \text{ mc.}$$

For a ferrite with $\Delta H = 200$, this would give $f \geq 4200$ mc. Actually both numbers are pessimistic since, for polycrystals, the absorption on the tail of the curve is usually smaller than that predicted by the Lorentzian line with the given half-width. For the numbers to be meaningful at all, the low field loss must be eliminated by making sure that the material is completely magnetized, or by reducing the saturation moment so that they do not occur.

ACKNOWLEDGMENT

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Reciprocal Ferrite Devices in TEM Mode Transmission Lines*

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Summary—**Several new reciprocal ferrite devices have been designed in TEM mode transmission lines to operate over both narrow and extremely broad bandwidths in the low-microwave frequency region. These include variable attenuators, an amplitude modulator, and a traveling-wave tube equalizer. Each component utilizes the attenuation associated with gyromagnetic resonance in low saturation magnetization ferrites. The techniques used to overcome the matching problems inherent in TEM mode transmission lines when ferrite loaded, and the design considerations pertinent to each component, are treated in detail. The parameters affecting the characteristics of each device are discussed and both final design and operating characteristics of the components are presented.**

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INTRODUCTION

IN recent years, considerable effort has been devoted to the design of a wide assortment of ferrite devices in various types of microwave transmission lines. Included among these are nonreciprocal ferrite